light, at an hundred foot diſtance, and that at an hundred and twenty foot diſtance I could diſcern ſome of the words. When I made this tryal, its Aperture (defined next the Eye) was equivalent to more than an inch and a third part of the Object-metall. This may be of ſome uſe to thoſe that ſhall endeavour any thing in *Reflexions* ; for hereby they will in ſome meaſure be enabled to judge of the goodneſs of their Inſtruments, &c.

N. B. The Reader may expect in the *next Month* another Letter, which came ſomewhat too late to be here inſerted ; containing a *Table*, calculated by the ſame Mr. *Newton*, about the ſeveral *Apertures* and *Charges* anſwering the ſeveral *Lengths* of theſe Teleſcopes.

EPITOME

Binæ Methodi *Tangentium* Doctoris *Johannis Walliſii* Geom. Prof. Saviliani *Oxoniæ* ; aliàs fuſius & explicatius ab ipſo traditæ, hîc verò ob anguſtiam loci compendifactæ : In quarum Schematiſmis ſi forſan literæ quædam redundaverint, illæ ad ea pertinere cenſendæ ſunt, quæ in ampliori ejuſdem Scripto continentur, hîc vero dictâ de cauſa omittantur.

H*Abes hic* (*Clariſſime vir*) *eorum ſummam* (*ſtrictim traditam*) *quæ fuſiùs ſcripſeram, meas de* Tangentibus Methodos *ſpectantia* ; *duas potiſſimum quibus præſertim utor* ; *alteram in Speciebus, alteram in Lineis* ; *utramque generali formâ facilè explicabilem.*

Priorem adhibeo Con.Sect.prop.23,30,36,46,49. *& paſſim alibi. Quæ hæc eſt.*

Expoſitâ Curvâ $A\alpha$, (*putà* Parabola, fig. 4.) *quam in* α *tangat* αF, *diametro* VDA *occurrens in* F ; *ordinatim applicentur* αV, *&* DOT *curvæ in* O *& tangenti in* T *occurrens. Ponatur autem* $V\alpha = \mathrm{b}$, $VA = \mathrm{v}$, $VF = \mathrm{f}$, $VD = \mathrm{a}$, *adeóque* $DA = \mathrm{v} \pm \mathrm{a}$, $DF = \mathrm{f} \pm \mathrm{a}$;

Eſt (*propter ſimilia triangula*) $VF.DF :: V\alpha.\ DT = \frac{\mathrm{f} \pm \mathrm{a}}{\mathrm{f}}\mathrm{b}$.

Item, ſi tangens ſit ultra curvam, $DT \geq DO$; *ſi citra,* $DT \leq DO$: *Nempe,* $DT = DO$ *ſi intelligatur* D *in* V ; *ſed, ſi extra* V, DT *vel* DO *major prout tangens eſt ultra citrave curvam.*

Tum,

Tum, habita ipſius DO deſignatione quæ ſit expoſitæ curvæ accommodata; (*puta, in Parabola, propter AV. AD :: Vaq. DOq*$=\frac{v \pm a}{v} b^2$; $DO = b\sqrt{\frac{v \pm a}{v}}$:) *fiat debita reductio*, (*puta, propter* $\frac{f \pm a}{f} b \overline{>} b\sqrt{\frac{v \pm a}{a}}$, *adeoque* $\frac{f^2 \pm 2fa + a^2}{f2} \overline{>} \frac{v \pm a}{v}$, & $f^2 v \pm 2fva + va^2 \overline{>} f^2 v \pm f^2 a$; *deletis utrinque æqualibus, hoc eſt, iis omnibus in quibus* a *non conſpicitur; cæteriſque per* $\pm a$ *diviſis* : $2fv \pm va \overline{>} f^2$.)

Tandem (*qui methodi nucleus eſt*) *poſito D in V*, (*quò ſit* $a = o$, *adeoque evaneſcant ipſius multipla omnia*,) *æquatio exhibebit* f *quæſitam* (*puta* $2fv \pm va = 2fv = f^2$, *adeoque* $2v = f$.)

Hanc (*locis citatis*) *accommodatam videas Parabolæ, Ellipſi, Circulove, Hyperbolæ, Paraboloidibus omnibus*, (*quibus & harum Reciprocas accenſeo*,) *atque alibi aliis*.

Ciſſoidi (fig.5.) *ſic accommodes. Eſt* (*per* cap. 5. pr.29. de Motu) $Va = b = \frac{v^2}{s = \sqrt{vh}}$, (*poſito* s *pro ſinu recto in circulo generante, cujus radius* r, *ſinus verſus* v, & $2r - v = h$, & $h - v = 2x$,) *adeoq*; (*ſubſtitutis* $v \pm a$ *pro* v, & $h \mp a$ *pro* h,) $\frac{v^2 \pm 2va + a^2}{\sqrt{:vh \pm 2xa - a^2:}}$ (= *D O*) $\overline{>}$ (*DT*$=\frac{f \pm a}{f} b =$) $\frac{f \pm a}{f} \times \frac{v^2}{\sqrt{vh}}$. *Ergo* (*ſumptis quadratis, & multiplicando decuſſatim*,) $f^2 v^5 h + 6f^2 v^3 ha^2 \dot{+} f^2 vha^4 \pm 4f^2 v^4 ha \pm 4f^2 v^2 ha^3 \overline{>} f^2 v^5 h \pm 2fv^5 ha \dot{+} v^5 ha^2 \dot{\pm} 2f^2 v^4 xa + 4fv^4 xa^2 \pm 2v^4 xa^3 - f^2 v^4 a^2 \mp 2fv^4 a^3 - v^4 a^4$: *item* (*deletis utrinque æqualibus, cæteriſque per* $\pm va$ *diviſis*) $\pm 6f^2 v^2 ha \pm f^2 ha^3 + 4f^2 v^3 h + 4f^2 vha^2 \overline{>} 2fv^4 h \pm v^4 ha + 2f^2 v^3 x \pm 4fv^3 xa \dot{+} 2v^3 xa^2 \mp f^2 v^3 a - 2fv^3 a^2 \mp v^3 a^3$. *Denique* (*poſito D in V, quo evaneſcat* a *cum ſuis multiplis, cæteriſque per* 2 fv^3 *diviſis*) *fiet æquatio* $2fh - fx = vh$, *adeoque* $\frac{vh = s}{2h - x = r + h = 3r - v} = f$.

Idem ſuccedet, ſumptâ, pro VA, diametro YA, (*cui tangens occurrat in* Φ) *aliâ* $\overset{}{\underset{ue}{YE}}$. *Item, ſi exponeretur curva quæ ordinatas non habeat, ſed quæ his æquipolleant; ut ſunt, in Spirali, creſcentes radii.*

Sed & calculi magna pars præverti poteſt; omiſſis ab initio (*utpote pòſt reijciendis*) *terminis iis ubi habetur* a^2 *vel ſuperior hujus poteſtas; item iis in quibus nec* a *conſpicitur, nec ſunt in* a *ducendi*, (*utpote æqualibus utrinque prodituris.*) *Exempli gratiâ*,

In

In Conchoide, (fig 6.) *cujus ordinata VMα conſtat ex ſinu recto VM* $= s = \sqrt{vh}$, *& tangente Mα* $= CH = \frac{s}{\kappa} r$, (*ſi ſit CP* $= CA = r$, *adeoque CH* $= AS$;) *ſaltem* $= \frac{s}{\kappa} \rho$ (*poſito CP* $= \varrho$:) *adeoque Vα* $= b = s + \frac{s}{\kappa} r = \frac{\kappa + r}{\kappa} s = \frac{h}{\kappa} s$, *ſaltem* $\frac{\kappa + \rho}{\kappa} s = \frac{n}{\kappa} s = \frac{n}{\kappa} \sqrt{vh}$ (*poſito* $\kappa + \varrho = n$.) *Ergo DT* $= \frac{f \pm a}{a} b = \frac{f \pm a}{f \kappa} n \sqrt{vh} \geqq DO = \frac{n \mp a}{\kappa \mp a} \sqrt{: vh \pm 2 \kappa a - :}$ (*omitto* a^2, *quia poſt delendum, indeque oriunda, & ſic ſemper* :) &, *ſumptis quadratis*, $\frac{f^2 \pm 2fa +}{f^2 \kappa^2} n^2 vh \geqq \frac{n^2 vh \pm 2 n^2 \kappa a \mp 2 n vha \pm}{\kappa^2 \mp 2 \kappa a -}$ (*hoc eſt*, $\overline{>}$ *ſupra, ſed* $\overline{<}$ *infra, punctum flexus contrarii.*) *Et, decuſſatim multiplicando ; omiſſis* (*ut præcipitur*) $f^2 \kappa^2 n^2 vh$ *utrobique , omnibuſque* a^2 *mnltiplis ; cæteriſque per* $\pm a$ *diviſis* ; $2 f n^2 vh \kappa^2 - 2 f^2 n^2 vh \kappa \pm \geqq 2 f^2 n^2 \kappa^3 - 2 f^2 n vh \kappa^2$: *adeoque* (*poſito D in V*,) $n vh \kappa = f n vh + f n \kappa^2 - f vh \kappa = f n r^2 - f vh \kappa$ (*propter* $vh + \kappa^2 = s^2 + \kappa^2 = r^2$,) & $f = \frac{vh = s^2}{n r^2 - vh \kappa} n \kappa$. *Et quidem, in primaria*, (*propter* $h = n$,) $f = \frac{s^2 \kappa}{r^2 - \kappa v}$.

In Figura Tangentium (fig.7.) *quæ à Conchoide differt, exempto quadrante genitore ; idem erit proceſſus, niſi quod, propter Vα* $= M\alpha = \frac{\rho}{\kappa} s$ (*non* $\frac{n}{\kappa} s$,) *prodibit* (*ſive in primaria, ſive in protracta contractave*,) $f = \frac{vh = s^2}{r^2} \kappa$.

In Figura Secantium (fig.8.) *propter Vα* $= b = \frac{r^2}{\kappa}$; *erit DO* $= \frac{r^2}{\kappa \mp a} \overline{>} \frac{f \pm a}{f \kappa} r^2 = DT$. *adeoque* $f = \kappa$.

Cumque hæc curva ſit Hyberbola (*per* pr.30.cap.5.& pr.1.cap. 15. de Motu,) *cujus Aſymptotæ CA, Cβ : eadem tangens habetur per* pr. 36.Con. ſect. *Cumque ordinatæ ad aſymptotas* (*per* pr. 94, 95, Arith. Infin.) *ſint* ſeries Reciproca Primanorum (*quæ ad Paraboloïdium genus ſpectat, verticem habens C, exponentem* -1,) *habetur eadem tangens per* prop. 49. Con. Sect. (*eademque eſt expedita methodus pro hyperbolæ cujuſvis tangente per aſymptotam inveniendâ.*) *Quippe, in Paraboloïdibus omnibus, ut intercepta diameter VC, ad VF, ſic* 1 *ad exponentem : hoc eſt, in præſenti caſu, ut* 1 *ad* -1 ; *adeoque VC*$=$*VF, ſed* (*propter contraria ſigna* $+$ $-$) *ad contrarias partes.*

Notandum hic ; in Parabolâ, Paraboloïde, Hyperbolâ, Ellipſi, &c. figurâve Sinuum (*rectorum, verſorumve,*) *Arcuum , Tangentium,*

gentium, Secantium, &c. aliâve cujus conſtructio eſt Similaris; protractio contractiove figuræ (ſeu mutatio Lateris-recti, aut quod hujus inſtar eſt,) non mutat punctum F, (eo quod Latus-rectum æquationem quæ longitudinem VF determinat non ingrediatur, utut eam ingrediatur quæ determinat longitudinem Vα, mutetque angulos ad α & F:) ſed ubi conſtructio eſt Diſſimilaris, ut in Cycloïde & Conchoïde (propter ordinatam illic ex Sinu & Arcu, hîc ex Sinu & Tangente, conflatam,) aliiſque iſtiuſmodi, res ſecus eſt: eò quòd una pars (ut Arcus in Cycloïde & Tangens in Conchoïde) protrahitur contrahiturve, manente alterâ (putà, in utriſque Sinu recto) ut in primariâ.

Idemque dicendum de Angulo applicationis (ad V,) cujus mutatio non mutat longitudinem VF, ſed neque Vα, quia neutrius ingreditur æquationem. Atque hinc fit, quòd in figura Scalena, quæ ordinatas contrarias, utrinque ad V poſitas, ſpectant tangentes, utut inæquales, in eodem F conveniant. Sed & (ut hoc obiter moneam) quadratorum aggregatum habent idem atque in erectâ; nempe ſemper $= 2\,V\alpha q + 2\,VF q$.

Eſtque hæc mihi methodus pro Maximis & Minimis *in omne genus quantitatibus.*

Methodus altera (ſecundùm tradita de Angulo Contactus & Arithm. Infin.) *curvam conſiderat tanquam ex particulis conflatam infinitè exiguis, ſed certam poſitionem habentibus; eandem nempe (propter contactûs angulum ſive nullius magnitudinis ſive infinitè exiguæ) cum recta ibidem tangente: adeoque cum hac (reſpectu cujuſvis rectæ) pariter declivem, (ut eſt Montis Aα* fig. 4, 5, *declivitas in α, eadem quæ tangentis α F.) Cujus ergo quæque particula (per* cap. 2. de Motu *) eſt in eâ ratione magis longa (quàm eſt reſpectiva expoſitæ rectæ particula æquè-alta) quâ eſt minùs declivis; puta αT quàm VD: Unde, propter mutatam in ſingulis punctis declivitatem, oritur ſeries longitudinum inæqualium in curvâ, ſeriei æqualium in rectâ, reſpondens; curvæ ad rectam rationem exhibens. Atque hinc methodus mea pro* curvis rectificandis, (ſchol. prop. 38. Ar. Infin. *inſinuata,) quam proſequor tractatu de* 'Ευθύνσει, *item* de motu cap. 5. prop. 13. & ſeqq. *Cujus aliqua pars eſt hæc de Tangentibus, ut quæ non totam declivitatum ſeriem perpendit, ſed eam quæ eſt in expoſito puncto.*

Hanc reſpectivam particularum longitudinem, aliàs inſinuatum *eunt (motu forinſecus aſſumpto) per motuum quibus tranſigantur* ἰσοχρόνων *celeritatem.* (*Quippe idem eſt, in Motu, Celeritas, atque hæc, in Situ (propter poſitionem obliquam ſeu minus declivem) reſpectiva* Longitudo.) *Aptiſſimè quidem in lineis a motu primitus oriundis, (putà, Cycloide, Conchoide, Spirali, Quadratrice, &c.) nec ineptè in aliis, quæ fingi ſaltem poſſunt iſtiuſmodi motibus deſcribi.*

Præſumo autem (ex prop. 15. cap. 2. de Motu) *eam eſſe curvæ in quovis puncto directionem, adeoque & declivitatem, quæ eſt rectæ ibidem tangentis: Item (ex* prop. 6. cap. 10.) *Motûs compoſiti directionem eſſe in Diagonio parallelogrammi, cujus latera & anguli exhibeant componentium celeritates & directiones.*

Intelligatur jam (fig. 4.) $A\alpha$ parabola, *deſcribi motu compoſito, ex æquabili ſecundum* $A\Upsilon$ *vel* $V\alpha$, *cujus itaque particulæ* ἰσόχρονοι (*per* pr. 3. cap. 10. de Motu) *ſunt ſeries Primanorum, quæ ad ſeriem totidem ultimæ æqualium, (hoc eſt, ad rectam* ἰσόχρονον *celeritate in* α *acquiſitâ tranſigendam,) eſt ut* 1 *ad* 2, (*per* Ar. Infin. pr. 64. *vel* pr. 1. cap. 5. de Motu.) *Adeoque, ſumpta* $VF = 2VA$, *& compoſito* $FV\alpha\upsilon$ *parallelogrammo; juncta* αF *eſt Tangens.*

Idem ſimiliter obtinebitur in Paraboloidibus quibuſcunque, ope prop. 2, 5, 6, 7, de Motu.

Atque inde facile (vel ex iiſdem principiis) oſtenditur; ſi intelligatur Fig. $A\Upsilon\upsilon$ *ſic conſtituta, ut momenta (reſpectu* AF) *ordinatorum* $\Upsilon\upsilon$, $y\omega$, *ſint ipſis* $\Upsilon\alpha$, yO, *ordinatis proportionalia; erunt Celeritates acquiſitæ in* α, o, *ſeu* V, D, (*poſitâ* $A\Upsilon$ *linea motûs æquabilis) rectis* $\Upsilon\overset{\upsilon}{\alpha}$, $y\omega$, *proportionales: Et conſequenter, ut* $A\upsilon\Upsilon$ (*illarum aggregatum) ad* $AF\upsilon\Upsilon$ (*aggregatum totidem maximæ æqualium.) ſic* VA (*aggregatum celeritatum ſeu particularum creſcentium) ad (aggregatum totidem maximæ æqualium)* ΥF.

Spiralis $AS\alpha$ (fig. 9.) *punctum* α *deſignatur motu compoſito ex recto per* $A\alpha$, *& circulari per* $V\alpha$, *æquabilibus utriſque &* ἰσοχρόνοις. *Ergo, ſumpta circuli tangente* $\alpha\omega = \alpha V$, *& completo* $A\alpha\omega F$ *parallelogrammo; juncta* αF *Spiralem tanget.*

Unde ſtatim emergit Archimedea quadratura (ſive Circuli ſive Sectoris cujuſvis) propter $AF = \alpha\omega = \alpha V$.

Sin motuum alter, puta $A\alpha$, *ſit acceleratus vel retardatus; pro*

A,

αA, ſumenda erit αB (in ea ad illum ratione quam illa poſtulat acceleratio ſeu retardatio,) eritque diagonium αβ, Tangens quæſita.

Quadratricis *AαB* (fig. 10.) *punctum α deſignatur motu compoſito ex recto per υα, & circulari in ϒα (æquabilibus &* ἰσοχρόνοις.) *Ergo, ſumpta tangente αV=αϒ, & completo parallelogrammo VωF, juncta αF tanget Quadratricem.*

Atque hinc alia quadratura, per Tangentem quadratricis, propter υF=αV=αϒ.

Illa per quadratricis Baſin, ſic elicitur. Poſitis CA=r, AQ=q, ϒZ=χ, QR=a. Erit (propter Quadratricis conſtructionem) AQ. RQ :: AC. aAE $= . \frac{a}{q} r :: \Upsilon Z . \alpha Z = \frac{a}{q} \chi$. *Eſtque αZ > αE ſumpto ubivis in AB puncto α, præterquam in B, quo caſu (evaneſcente utraque) erit αZ=αE, adeoque χ=r; hoc eſt, ϒZ=XB =AC. Sed & υE communis tangens utrique curvæ XB, AB.*

Cycloïdis (fig. 11.) *punctum α deſcribitur motu compoſito, ex recto in αV, & circulari in αβ (æquabilibus & æquè-velocibus.) Ergo, ſumpta tangente αω=αV, & completo VωF parallelogrammo, junctа αF Cycloidem tanget. Et quidem, propter Ang. υαF (=αβF* $= \frac{1}{2}$ *αCF)* $= \frac{1}{2}$ *υαV, occurret circuli αβ erectæ diametro in vertice.*

In ſecundariis (contractâ protractâve) ſumenda erit αω ad αV, in ea ratione major minorve, quâ eſt celeritas motûs circularis ad celeritatem recti.

In Figura Arcuum, Sinuumve, (fig. 12.) *procedendum ut in Cycloide, niſi quòd (propter exemptum ſemicirculum genitorem) pro tangente αω illic (quæ hîc eſt ατ) ſumenda erit erecta αω æquealta.*

Conchoidis (fig. 6.) *punctum α deſignatur motu compoſito, ex æquabili circulari in αβ (hujuſve tangente αω) & recto in αϒ accelerato pro incremento tangentium: quæ quidem acceleratio duplex eſt, altera propter declivitatis angulum βαϒ, hoc eſt, υαϒ, continuè creſcentem; altera propter radii in ſecantem protractionem, continuè item creſcentem. Propter priorem, ducta tangente αυ (quæ occurrat in υ regulæ CH,) recta υζ (parallela rectæ PHα,) occurrat αϒ in ζ: Propter poſteriorem; eadem υζ protracta occurrat tangenti verticis in Z: indeque Zϒ rectæ υαX parallela; adeoque αϒ=XZ.*

$\alpha\zeta :: CM . $ ~~M~~ $S :: P\mu . PH$. *Completo denique* $\Upsilon\alpha\upsilon f$ *parallelogrammo, juncta* αf *tanget conchoidem.*

In ſecundariis (ubi non eſt $CP=CA$*,) ſumenda erit* $\alpha\Upsilon$ *ad jam deſignatam, ut eſt* CP *ad* CA.

In Figura Tangentium (fig. 7.) *propter exemptum Conchoidi quadrantem genitorem, pro tangente* $\alpha\upsilon$ *illic (quæ hîc eſt* $\alpha\tau$*) ſumenda erit erecta* $\alpha\upsilon$ *æquè-alta.*

Pluribus exemplis proferendis ſuperſedeo. Moneo tamen, utramvis Methodum, utut tangentibus rectis hîc accommodatam, extendi poſſe ad mutuum Curvarum tactum. Puta; ſi, pro $FV\alpha$ *triangulo* (fig. 4,5,) *intelligatur Hyperbola; recta DT, quæ hîc inſignitur charactere qui triangulo convenit. ſubire tum debebit characterem Hyperboles; cujus vertex F ſimili proceſſu quæratur. Similiſque in poſteriori methodo accommodatus eſt linearum ductus. Et quidem, cum curvam* $A\alpha$ *tangens recta* αF*, ſit etiam tangens communis curvarum omnium, expoſitam ibidem tangentium; prout hic, ex data* $A\alpha$ *curva quæritur recta* αF*, ſic ex hac datâ (per eandem methodum inverſam) quærenda erit alia tangens curva, modò ſatis ſit determinata.*

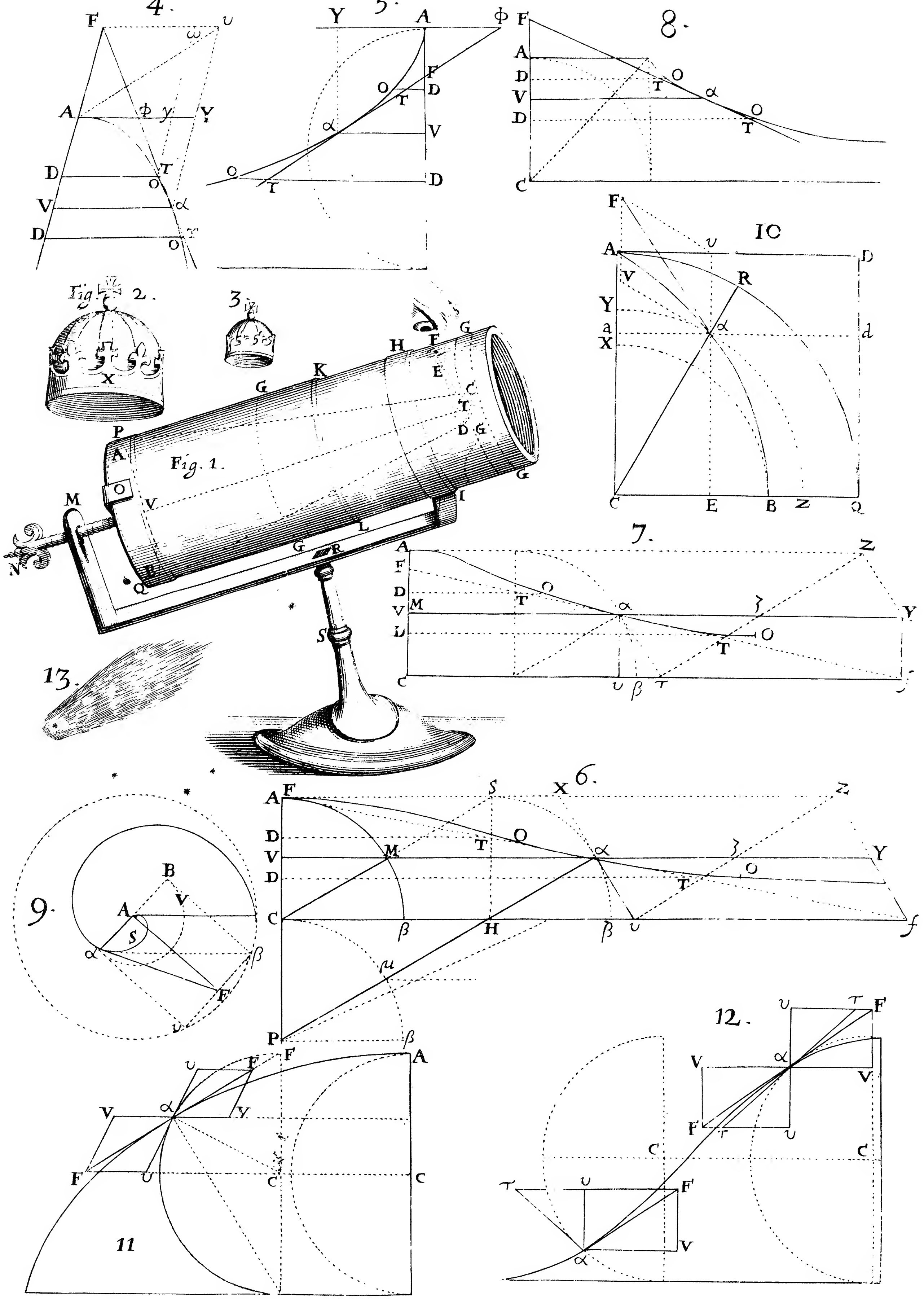
Sed ampliandum non eſt. Tu itaque Vale.

Tuus

Oxoniæ *die* 15.
Febr. 1671.

Johannes Wallis.

Extract


4.
F υ ω A Φ y Y D T O V α D T O
5.
Y A Φ F D O T α V o T D
8.
F A D V D C O T α O T
10
F A υ D V R Y a α d X C E B Z Q
Fig. 2.
X
3.
Fig. 1.
P A O V M N B Q G K H F E G C T D G G I L G R S
7.
A F D M V L C Z O T α ζ Y O T υ β τ f
13.
6.
A F S X Z D O T V M α ζ Y D T O C β H β υ f μ P β
9.
B A V S C α β F υ
F A υ F V α Y F υ C C
11
12.
υ τ F V α V F τ υ C C τ υ F V α